EE582 Spring 2019 Lab#1 Report

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Contents

1 Problem 1: Parameters Definition

1.1 Define a

The parameter a describes the time scale of the recovery variable u. Smaller values result in slower recovery. A typical value is a = 0.02[1].

1.2 Define b

The parameter b describes the sensitivity of the recovery variable $u$to the sub-threshold fluctuations of the membrane potential v. Greater values couple v and u more strongly resulting in possible sub-threshold oscillations and low-threshold spiking dynamics. A typical value is b = 0.2[1].

1.3 Define c

The parameter c describes the after-spike reset value of the membrane potential v caused by the fast high-threshold K^+ conductance. A typical value is c = -65mV[1].

1.4 Define d

The parameter d describes after-spike reset of the recovery variable u caused by slow high-threshold Na^+ and K^+ conductance. A typical value is d = 2[1].

1.5 Define u

The variable u represents a membrane recovery variable, which accounts for the activation of K^+ ionic currents and inactivation of Na^+ ionic currents, and it provides negative feedback to v[1], [2].

1.6 Define v

The variable v represents the membrane potential of the neuron. And v' denotes its derivative with respect to time. The variable v has mV scale and the time has ms scale[1], [2].

1.7 Define I

The variable I deliverers synaptic currents or injected dc-currents[1].

2 Problem 2: Phase Portrait

A phase portrait is a geometric representation of the trajectories of a dynamical system in the phase plane. A phase portrait graph of a dynamical system depicts the system's trajectories (with arrows) and stable steady states (with dots) and unstable steady states (with circles) in a state space. The axes are of state variables[3].

For a systems described by a bunch of linear differential equations as $x = Ax$. Its phase portrait is a representative set of its solutions, plotted as parametric curves (with t as the parameter) on the Cartesian plane tracing the path of each particular solution $(x, y) = (x_1(t), x_2(t)), -\infty < t < \infty$[4].

In this situation, the phase portrait shows how recovery variable u changes with membrane potential v as time.

3 Problem 3: Simulation Results & Analysis

3.1 Simulation Results

3.1.1 Screen Shots of Results

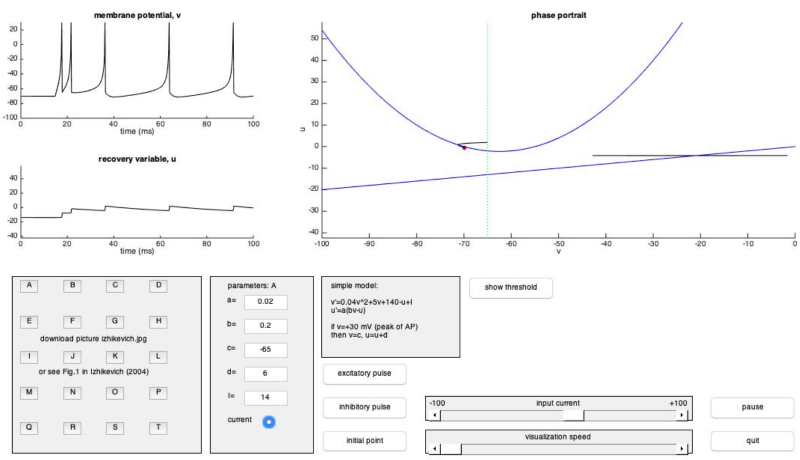

Figure 1: Simulation Result of A (Tonic Spiking)

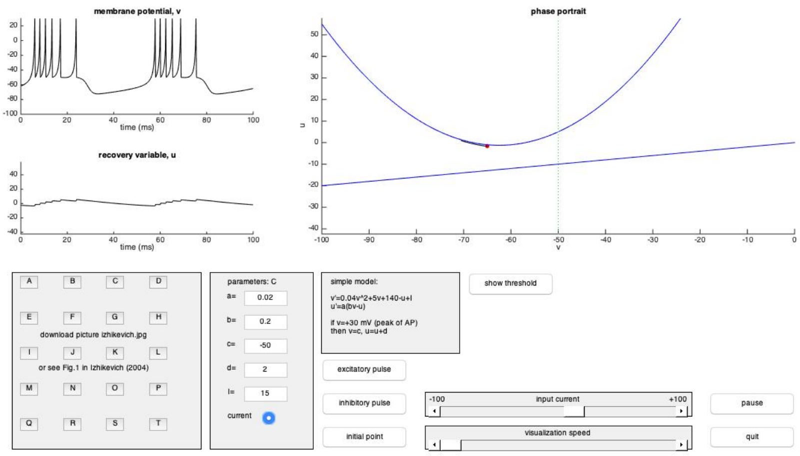

Figure 2: Simulation Result of C (Tonic Bursting)

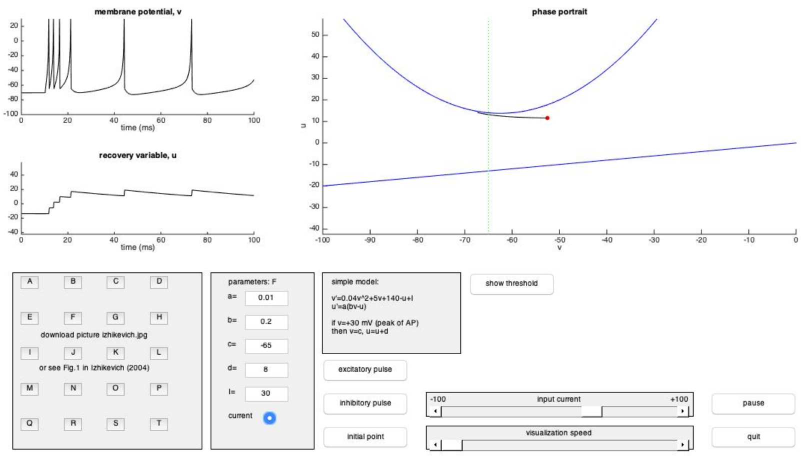

Figure 3: Simulation Result of F (Spike Frequency Adaptation)

Figure 1, 2 and 3 show the simulation result of each case. For tonic spiking case, the membrane potential v fires with a period around 30 ms. For tonic bursting case, the the membrane potential v fires with a period around 55 ms globally, but locally, it oscillates with a period as 4 ms. For spike frequency adaptation case, the neuron starts oscillating in a higher frequency than normal tonic spiking case, but after a while, it behaves the same as the tonic spiking case.

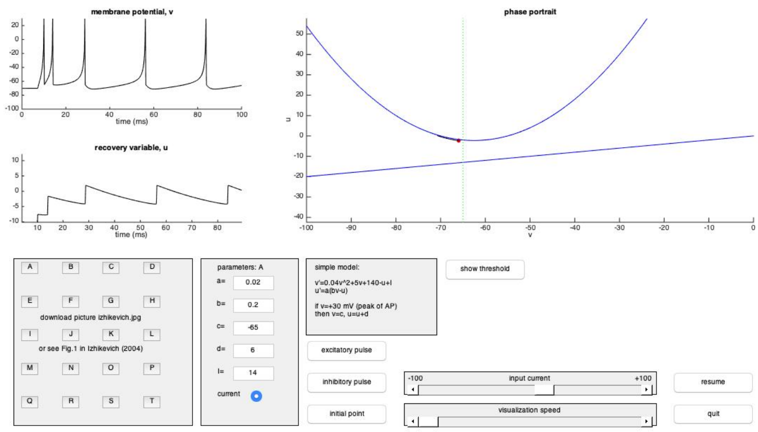

Figure 4: Zoom in Result of A (Tonic Spiking)

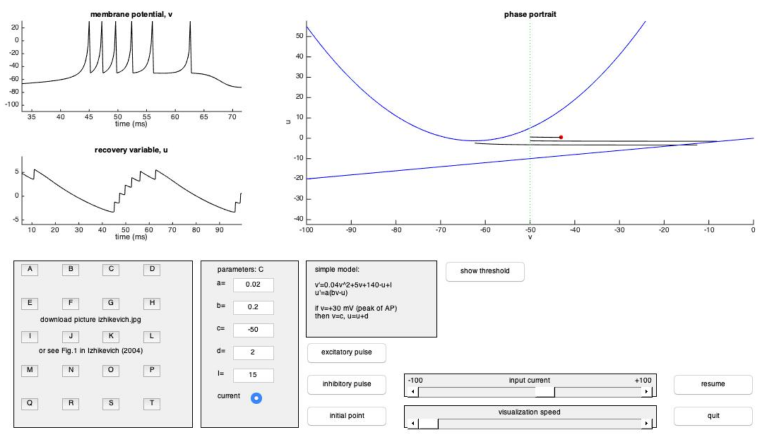

Figure 5: Zoom in Result of C (Tonic Bursting)

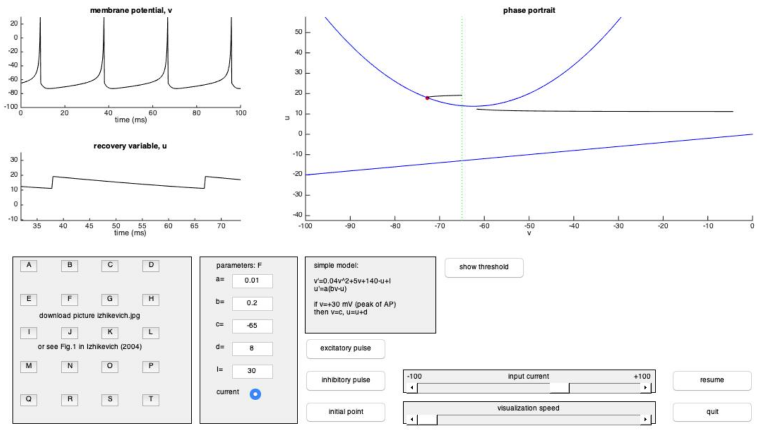

Figure 6: Zoom in Result of F (Spike Frequency Adaptation)

3.1.2 Comparisons between recovery variable (u)

Figure 4, 5 and 6 show the zoom-in result of recovery variable u of each case.

For case A (tonic spiking), the recovery variable u is a serrated shape function, and its amplitude is ranged from -4 to 3.

For case C (tonic bursting), although u is also serrated shaped, however, the trend of the function is kind of different with case A. For case A, the rising edge is a vertical line, like a step function, but for case C, the rising edge is like a ramp function. Secondly, the period of u is about 55 ms, which is the same with the period of the membrane potential v in case C.

For case F (spike frequency adaptation), the recovery variable u is also a serrated shape function, there are two prominent differences between case C and case A. First one is that the amplitude of case A is between -4 and 3, while the amplitude of case C is between 10 to 20. Second difference is that the rising of edge of case C is also like a ramp function, but it's not as obvious as in case C.

3.2 Membrane Potential (v) and Recovery Variable (u) vs. Input Current (I)

For this section, take the tonic spiking as an example.

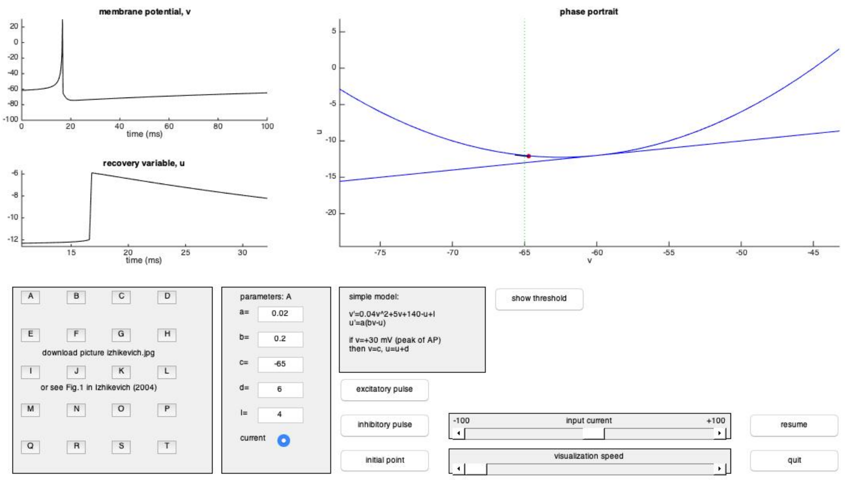

Figure 7: Simulation Result of case A when Input Current I = 4 mA

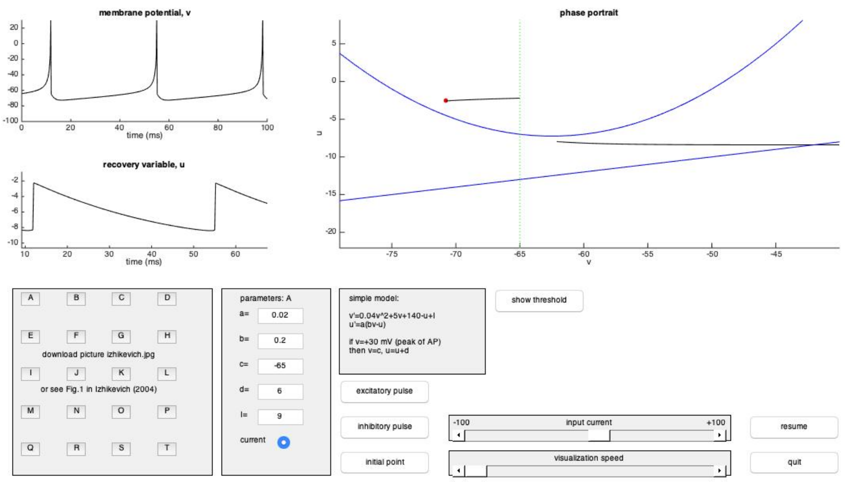

Figure 8: Simulation Result of case A when Input Current I = 9 mA

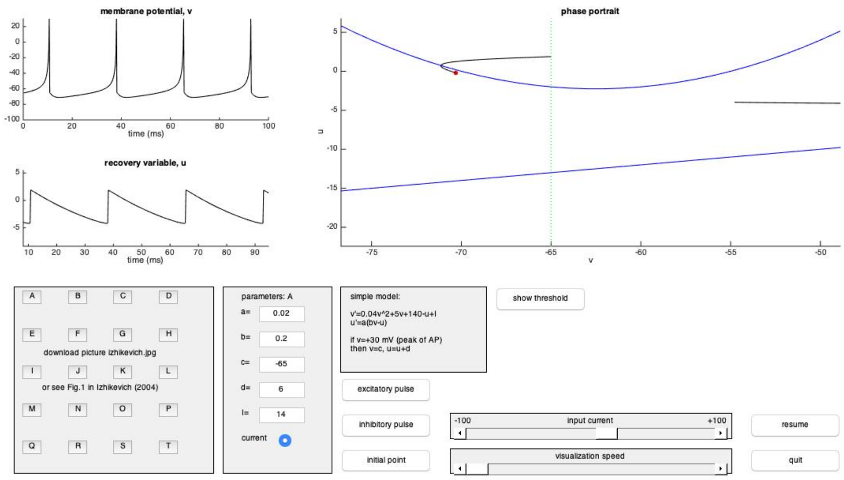

Figure 9: Simulation Result of case A when Input Current I = 14 mA

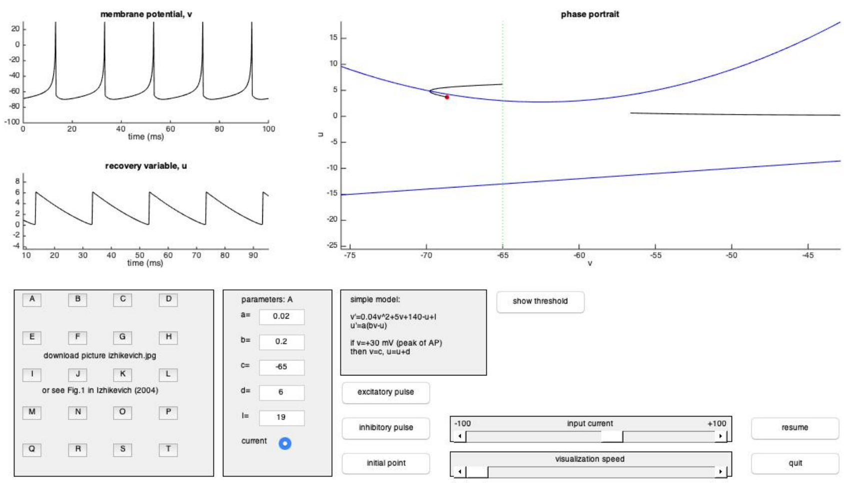

Figure 10: Simulation Result of case A when Input Current I = 19 mA

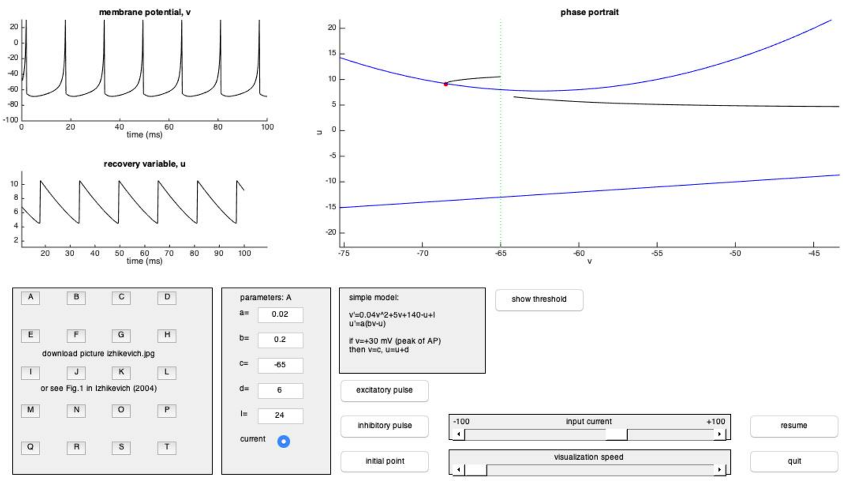

Figure 11: Simulation Result of case A when Input Current I = 24 mA

From Figure 7 to 11, we can see that the change of input current I will result in the change of the recovery variable u and the membrane potential v in both amplitude and period.

For example, in case I = 9 mA, the period of u and v is about 45 ms, while in case I = 14 mA, the period becomes 30 ms. And for amplitude, it is about -8 to -2 in i = 9 mA case, and in I = 14 mA case, it is between -4 to 2. And from the phase portrait, the distance between the parabolic lines and the linear function is getting closer as the input current I decreases.

Table 1: Amplitude & Period of u and v with Different I

I (mA)	Range of u	Period of u & v (ms)	Distance of Two Curves
4	-12 $\sim$ -6	>100	1
9	-8 $\sim$ -2	45	7
14	-4 $\sim$ 2	30	10
19	0 $\sim$ 6	20	15
24	4 $\sim$ 11	15	20

From Table 1, the conclusion is 1) The range of the recover variable u will be lifted as the input current I increases, but the difference of the upper bound and lower bound stay almost the same. 2) The period of u and v changes almost proportional to the distance of two curves in the phase portrait. 3) The distance of two curves is linear with the input current I.

3.3 Recovery Variable (u) vs. Parameters a & d in Tonic Spiking Case

To ensure the simulation is still in tonic spiking case, the parameter a and d cannot be changed too much, otherwise it's not in tonic spiking condition.

3.3.1 Parameter a Simulation Result

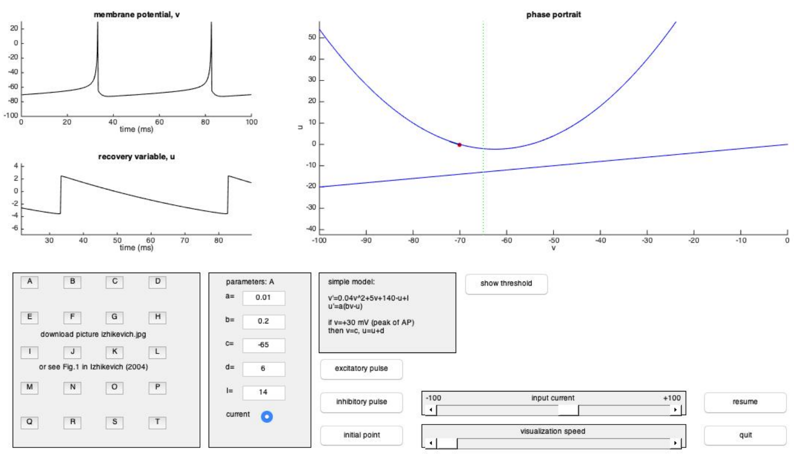

Figure 12: Simulation Result of case A when $a = 0.01$

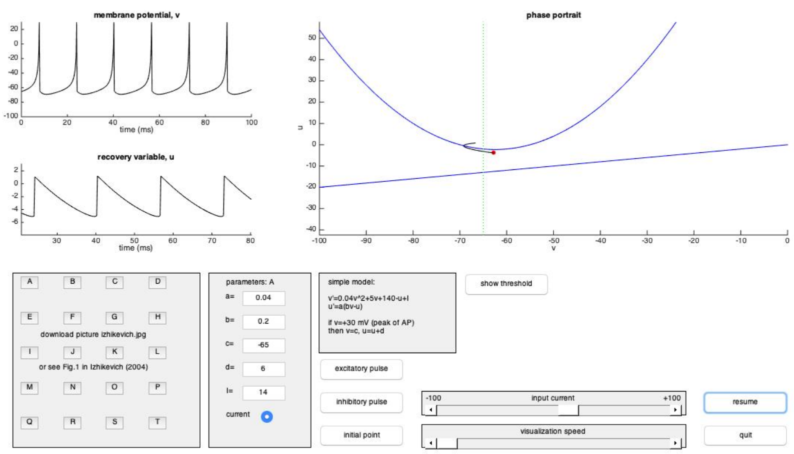

Figure 13: Simulation Result of case A when $a = 0.04$

Figure 12 and 13 show the simulation result when parameter a changes. The range of u will slightly change and the period will have an obvious change.

3.3.2 Parameter d Simulation Result

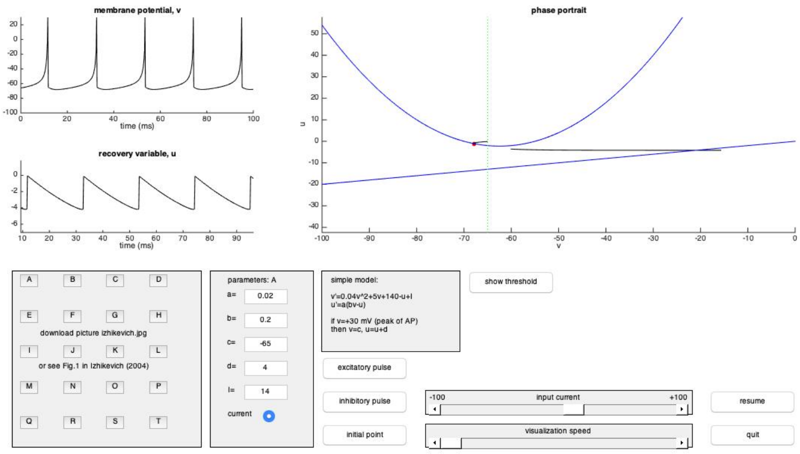

Figure 14: Simulation Result of case A when $d = 4$

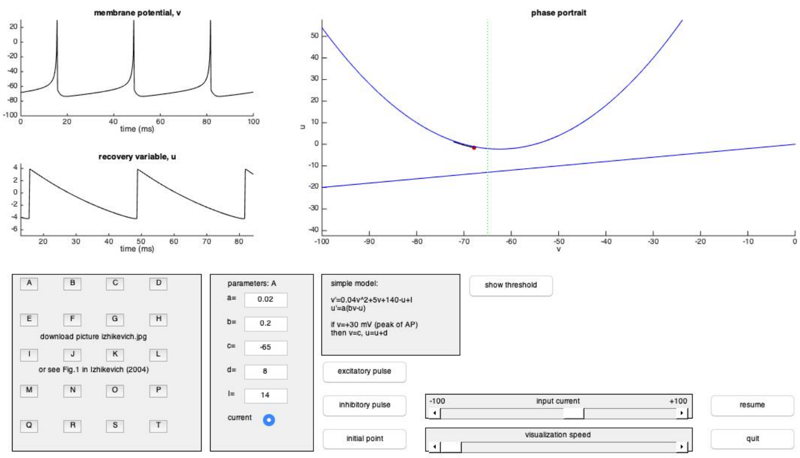

Figure 15: Simulation Result of case A when $d = 8$

Figure 14 and 15 show the simulation result when parameter d changes. The range of u and the period will have change obviously as d changes.

3.3.3 Conclusion

Table 2: Amplitude & Period of u and v with Different I

Case	**Range of u**	**Period of u (ms)**
$a = 0.01$, $d = 6$	-5 $\sim$ 1	18
$a = 0.04$, $d = 6$	-4 $\sim$ 2	50
$a = 0.02$, $d = 6$	-4 $\sim$ 2	30
$a = 0.02$, $d = 4$	-4 $\sim$ 0	20
$a = 0.02$, $d = 8$	-4 $\sim$ 4	35

From Table 2, the conclusion is 1) Change of a will slightly lift the range of u and change the period dramatically. 2) Change of d will lead to the change of upper bound of u, and slightly change the period.

3.4 Spikes Generated by Excitatory Pulse

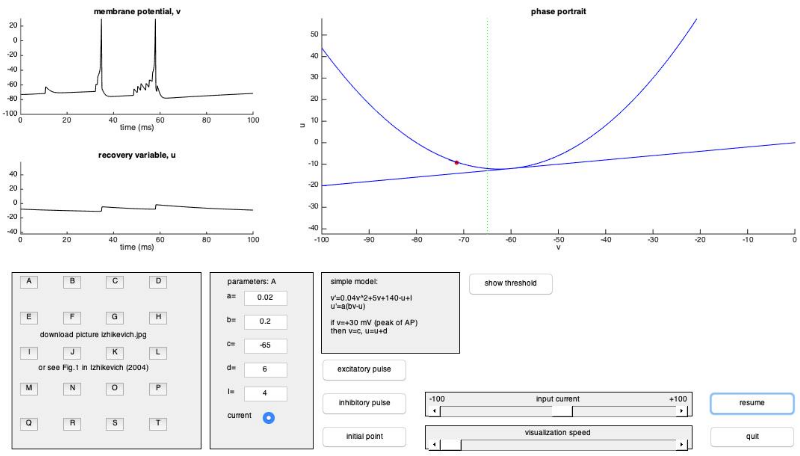

Figure 16: Simulation Result of Excitatory Pulse in Case A

Figure 16 shows the simulation result in case A when an excitatory pulse is given. The parameter I is chosen as 4 mA so that the period of spike is quite large and there is no self-generated spike in the graph. From the figure, we can see that when an excitatory pulse is given, and only when the amplitude of the pulse is large enough, the spike could be generated, otherwise the pulse won't generate the spike.

3.5 Spikes Inhibited by Inhibitory Pulse

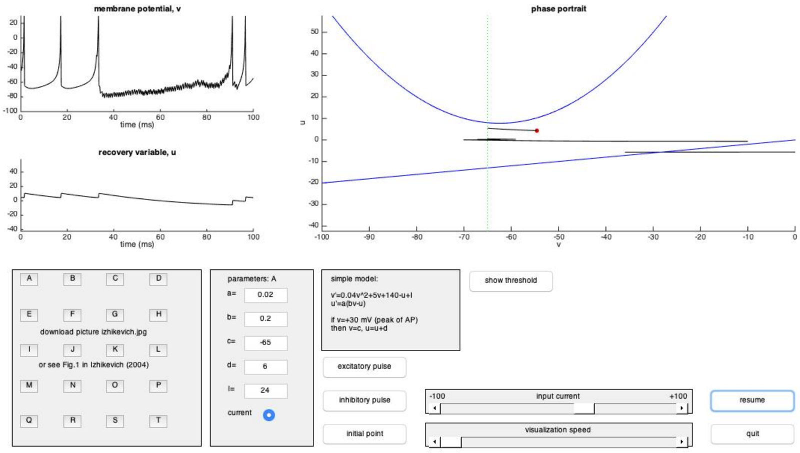

Figure 17: Simulation Result of Inhibitory Pulse in Case A

Figure 17 shows the simulation result in case A when an inhibitory pulse is given. The parameter I is chosen as 14 mA so that the period of spike is about 30 ms. From the figure, we can see that when continuous inhibitory pulses are given, it will slow down the rate the self-generation of spikes, but it can only delay the spike, but not eliminate it. And right after the delayed spike, next spike will be generated with a higher frequency.

3.6 Relationship between Model and Parabolic and Linear Functions in Tonic Spiking Case

The model can be described as three equations[1]:

$$
\begin{aligned}
v' &= 0.04v^2 + 5v + 140 - u + I \\
u' &= a(bv - u) \\
ifv \geq\ & 30mV => \ v = c,\ u = u + d
\end{aligned}
$$

The parabolic function in the phase portrait is:

$$u_1 = 0.04v^2 + 5v + 140 - u + I \quad when\ u \ = \ 0$$

And the linear function in the phase portrait is:

$$u_2 = a(bv - u) \quad when\ u \ = \ 0$$

As mentioned in section 2, the curve on the phase portrait is the solution of differential equation. For initial state, the value of u and v should be 0, then it will have a trend to move to the parabolic curve, and when they touched, since the intersection is a meta-stable point, it will move to the strait line immediately, and when it cross the linear line, the spike is generated, and keep doing this process periodically.

References

1. Eugene M. Izhikevich. Simple model of spiking neurons. *IEEE transactions on neural networks*, 14 6:1569–72, 2003.
2. E. M. Izhikevich. Which model to use for cortical spiking neurons? *IEEE Transactions on Neural Networks*, 15(5):1063–1070, Sep. 2004.
3. Wikipedia. Phase portrait, 2018.
4. Zachary S Tseng. The phase plane, 2008.