“Terahertz Opto-Electronics” Final Project Report

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Project Introduction

Given some details about the relationship between the chemical potential μ and the doping concentration N and the relationship between an auxiliary value $\prod_{02}$ and the doping concentration N, plot it directly with some integrating techniques in any compiler.

Solution Strategy

As the problem given, the chemical potential μ and the doping concentration N has a relationship as the equation:

$$N = -\frac{1}{\pi^2 l_B^2 \Gamma}\sqrt{\frac{\pi}{2}}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty} d\varepsilon \frac{1}{1+\exp(\frac{\varepsilon-\mu}{k_B T_e})} exp\left[-\frac{(\varepsilon-\varepsilon_n)^2}{2\Gamma^2}\right]$$

The key point is to simplify the integration term and find effective Landau level n.

For function $f(\varepsilon) = \frac{1}{1+\exp(\frac{\varepsilon-\mu}{k_B T_e})} exp\left[-\frac{(\varepsilon-\varepsilon_n)^2}{2\Gamma^2}\right]$, the value of f has some special properties. When the value of ε is very small or very large, the value of f is close to 0 obviously. Only when the value of ε is close to ε_n or μ, the value of f can be some efficacious value. When μ is not concerned (μ is very smaller or very larger than ε_n), the plot of $f(\varepsilon)$ looks like a pulse function centered at ε_n. For n=1, the plot of $f(\varepsilon)$ is shown below.

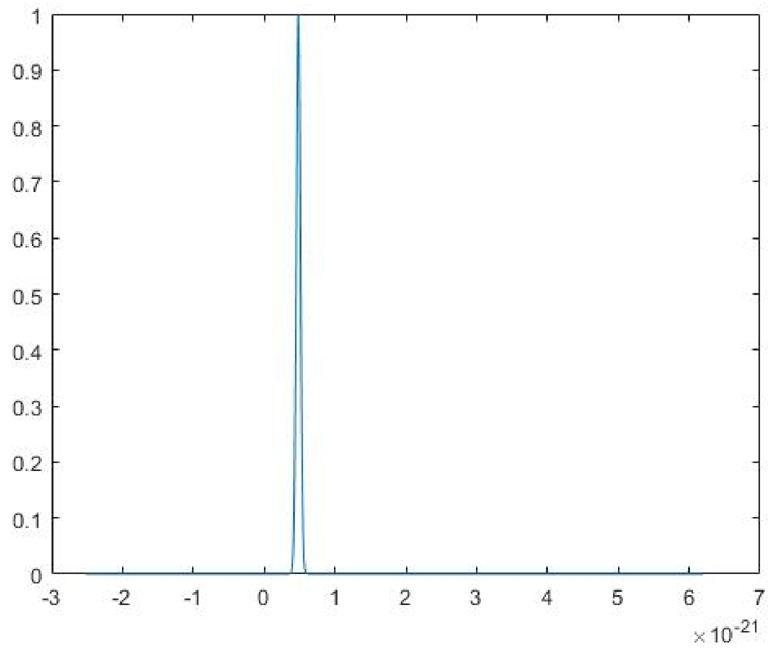

Figure: Plot of f(ε)

This figure shows that if one has taken an appropriate range of ε, the integration can be obtained easily. In my code, since the value of ε_n is around 1e-22 level, I take the range of ε as 6e-21, as 3e-21 in both positive direction and negative direction, as:

$$\varepsilon \in [\varepsilon_n - 3 \times 10^{-21}, \varepsilon_n + 3 \times 10^{-21}]$$

When μ is also concerned, I made some compromise in choosing the range of ε. The range of ε is:

$$\varepsilon \in [\min(\varepsilon_n, \mu) - 3 \times 10^{-21}, \max(\varepsilon_n, \mu) + 3 \times 10^{-21}]$$

The Integration can be substituted by addition with a step as 2.5e-24. The next key point is to find an appropriate n. We can find that if the value of μ is too larger or too small than ε_n, the integration of f(ε) is a small value. My strategy is keep trying until get a proper n, beyond which the value of integration of f(ε) is almost equal to zero. Table below shows the corresponding n to the magnetic field B:

B(T)	3.5	1.5	0.5	0.05
n	10	30	50	300

Table: n-B Relation (empirical result)

For small magnetic field B, we can see that it will lead to a larger n, it is because the value of ε_n is result from the value of ω_c, which is infected by the value of B. For smaller B, the value of ω_c is very small, so it need a large n so that it can fit the value of μ.

The relationship between chemical potential μ and doping concentration is showed below,μ is in meV level.

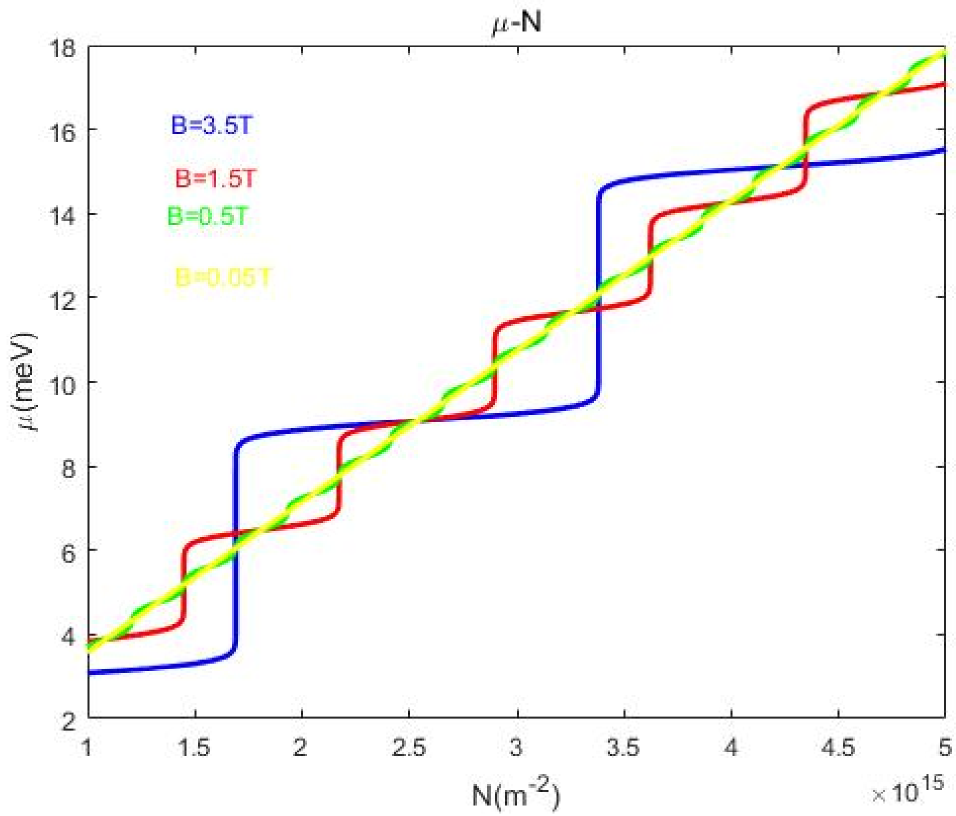

Figure: Plot of μ-N

The blue curve is corresponding to B=3.5T, the red one is B=1.5T, the green one is B=0.5T and the yellow one is B=0.05T. It is easy to find that for large B, the curve shows like a step function while when B is small, the step is dented.

For problem two, it introduces two landau levels n and n', so it will lead to two landau energies ε_n and $\varepsilon_{n'}$. The strategy in choosing the range of ε is the same, the range of it is:

$$\varepsilon \in [\min(\varepsilon_n, \mu, \varepsilon_{n'}) - 3 \times 10^{-21}, \max(\varepsilon_n, \mu, \varepsilon_{n'}) + 3 \times 10^{-21}]$$

and the step is equal to 1e-24.

As the problem gives, the value of Π_{02} is related to the term of

$$\sum_{n,n'=0}^{\infty} C_{n,n'}(l_B^2 q_\parallel^2/2)\Pi_2(n,n',\Omega)$$

This term is somewhat like a two-dimensional integration, since the position of n and n' is kind of reciprocal, my strategy is to set a maximum n_{max}, then n and n' will travel from 0 to n_{max}. Following table shows the relationship of magnetic field B and n_{max}.

B(T)	3.5	1.5	0.5	0.05
n_{max}	10	10	30	80

Table: n_{max}-B Relation (empirical result)

However, this will take too much time to calculate the $L_{n_2}^{n_1-n_2}$, which has a factorial term. Also by some trials, I found that when the difference between n and n' is very large, let's say $|n - n'| > 20$, the value of L is almost equal to 0. This strategy will help to save a lot of time for n is very large.

The result is showed below:

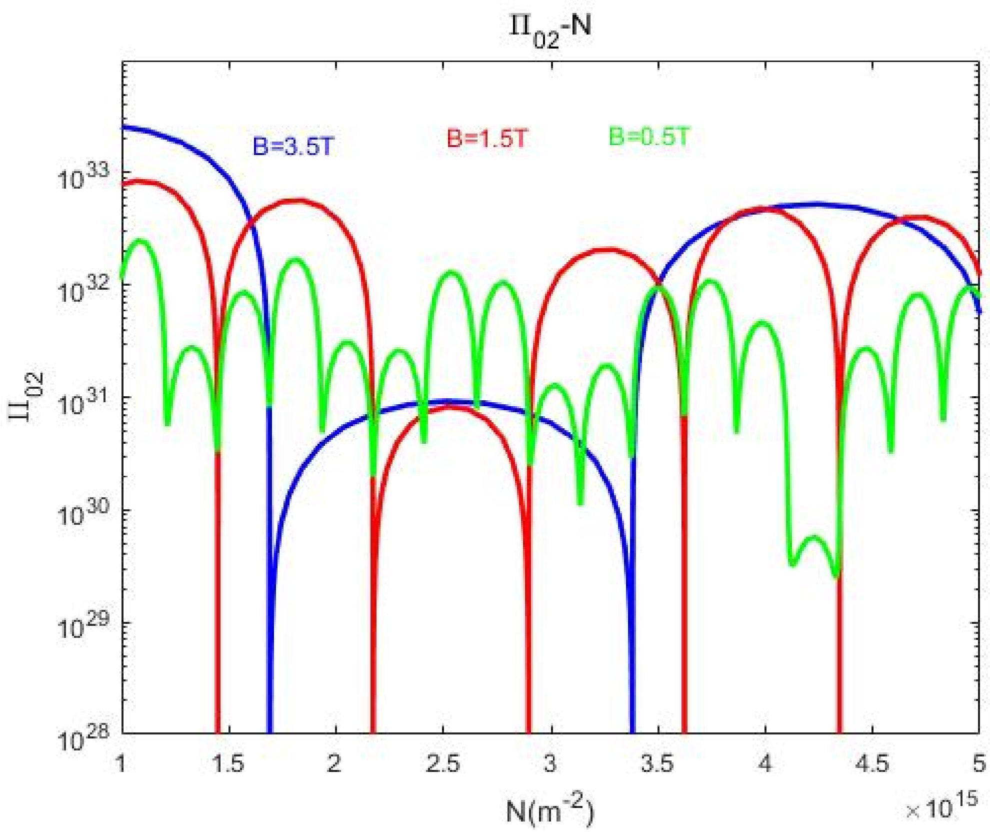

Figure: Plot of Π_{02}-N

In this figure, the case of B=0.05T is not showed since it is a little bit strange. The blue curve is corresponding to B=3.5T, the red one is B=1.5T and the green one is B=0.5T. It is easy to find that for large B, the curve will fluctuate in larger amplitude but a low frequency while for small B, the curve will fluctuate with a higher frequency.

Appendix

This figure shows the plot of Π_{02}-N with a magnetic B=0.05T. The strangest thing is that when n_{max} is taken a large value, which should be a good thing because it will lead to a more exact result, the result is calculated as NaN, which is not make sense. In this figure, n_{max} is taken as 80. However, N cannot reach 3e15 at all.

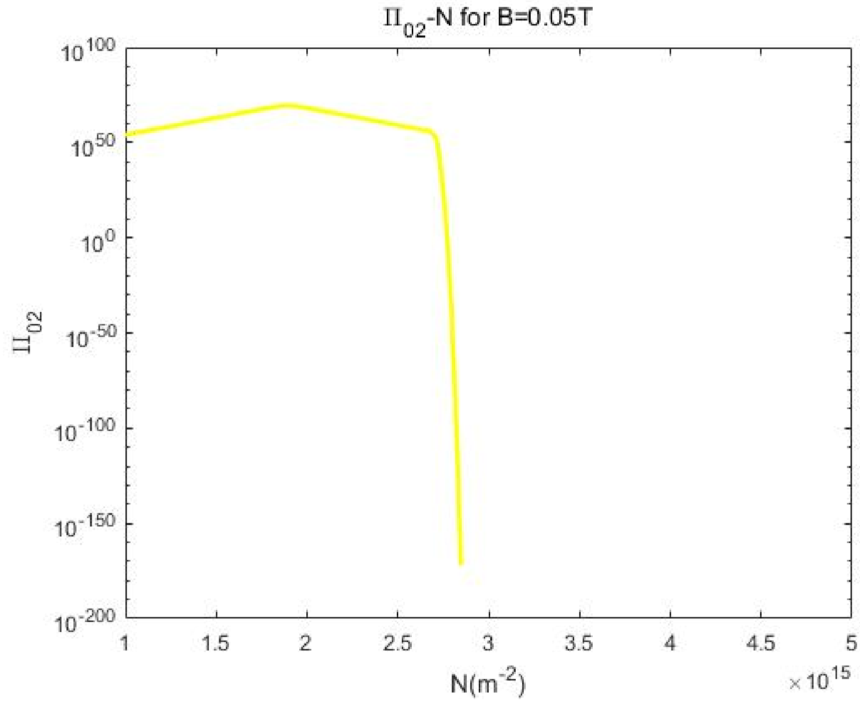

Figure: Plot of Π_{02}-N for B=0.05T

The whole program will run for 5 minutes.